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Processing seismic data representing a physical system

The present invention relates to processing seismic data representing a physical system. Such processing may be used in data inversion and a particular example of this is the

5   inversion of time-lapse seismic data.

Seismic reflection is a technique used to determine details of structures beneath the surface of the Earth. The resolution that may be achieved makes this technique the method of choice for oil exploration and mapping of subsurface rock structures. It is

10   also applicable to experimental research that probes the fine structure within the Earth's crust and at the crust-mantle boundary.

The technique involves generating downward-propagating seismic waves in succession at a number of locations within the region being explored. A large number of receivers

15   are positioned at intervals away from each source location and these receivers record the amplitudes (for example, in terms of pressure, displacement or its derivative) of seismic waves reflected back up to the surface from subsurface inhomogeneities over a period of time. The recorded waves are usually deconvolved, removing the effects of the source and receiver (which have their own response functions).

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Reflection data typically have low amplitudes and are contaminated by multiple reflections and other kinds of noise. Various acquisition and processing techniques may be used to improve signal-to-noise ratios, such as averaging (stacking) of traces with the same midpoint, taking into account different distances between source and receiver, and

25   discrimination of multiple reflections based on either their periodicity or wavefront angles which differ from the primary reflections. Further, the data may be correctly positioned in space by a process called migration, which moves dipping events into their correct position. When comparisons are made between two or more datasets over the same area, careful analysis between the amplitude, time and other attributes of the

30   datasets may be made.

After the appropriate corrections, which may further include correction for other known environmental variables, the data are combined to provide a graphical representation of the subsurface inhomogeneities.

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Seismic reflection data obtained by field experiments are then processed to obtain a three dimensional image of subsurface structures as described above. The three dimensions refer to the spatial dimensions "illuminated" by the seismic data. The vertical axis may represent depth or two-way vertical seismic wave travel time.

The amplitudes of reflected seismic waves are indicative of the subsurface reflection strengths, contaminated by noise. The reflection strength depends upon the reflection coefficient, which may be defined as a function of the relative contrasts of the elastic material properties of the subsurface layers.

The elastic properties of an isotropic, elastic medium are completely described by three parameters, for example the two elastic Lamé parameters and the density. Other parameterisations are possible, for example acoustic impedance, shear impedance and density. A third example is P-wave velocity, S-wave velocity, and density. The transformation between different sets of elastic parameters is well defined and straightforward.

In general, the elastic properties vary spatially. In order to explain the relationship between the elastic properties and the seismic data it may be convenient to imagine the subsurface as a stack of geological layers. The layer properties are described by the elastic properties of the rocks within the layers while the seismic data are related to the contrasts of the layer properties between successive layers. The seismic data are therefore suitable for interpreting subsurface layer structures since they image the boundaries between the layers.

Seismic inversion is defined herein as the process of transforming (inverting) seismic reflection data to elastic material properties, i.e. taking amplitudes (measurements of contrasts) and using them to infer physical layer properties. Numerous different seismic inversion techniques are known.

Over a period of time, certain types of rock, known as source rocks, will produce hydrocarbons. The produced hydrocarbons are then transferred to and stored in rocks known as reservoir rocks through various geological processes. During production of

hydrocarbons in a subsurface region, the effective elastic material properties of the reservoir rocks change with production time, where production time is the fourth dimension in seismic 4D analysis. The changes of the effective elastic properties of the reservoir rocks may be caused by changes of the pore fluid saturations in the reservoir

5   rocks, but also by pressure and temperature changes. Explained by a simple layer-based earth model concept, the properties of the reservoir layer are changed during production, implying changes in the reflectivity for the upper and lower reservoir interfaces. The measurements taken at a further seismic survey are related to the new contrasts at the boundaries between adjacent layers.

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Reservoir changes are often inferred from a comparison of the seismic data (e.g. amplitudes of seismic waves reflected at interfaces bounding or within the reservoir) for different seismic surveys acquired at different stages of the production. A more direct interpretation can be based on difference data. Difference data are established by

15   subtracting two time-separated seismic surveys covering a common part of the earth. The difference data, after the proper time-alignment during pre-processing, represent a spatial image of the changes of the relative contrasts between the two different acquisition times.

20   For a three dimensional seismic dataset, the classic inversion problem is to estimate the elastic material parameters from the three dimensional seismic data. A natural extension of 3D inversion to inversion of time-lapse seismic data (4D) is to invert the different 3D datasets separately by a known method, and then subtract the results to obtain the changes.

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However, the reliability of 4D interpretations is difficult to assess, and are made by qualitative assessment. A full consideration of the uncertainties involved is important for making an accurate inference of the changes in the reservoir properties between the two seismic surveys. The results of such seismic analysis may be important in reservoir

30   management in that the inferred reservoir properties are used to evaluate, for example, new drilling targets and future drainage strategies.

Seismic inversion provides quantitative estimates of the elastic reservoir properties. However, inversion of noisy seismic data is known to be a difficult and ill-posed

procedure. An appropriate assessment of the uncertainties in 4D inversion data has not previously been possible.

Commercial time-lapse inversion techniques have become available, but only with brief
5   descriptions of the methods. Some results have been published (Mesdag et al, 2003, Integrated AVO reservoir characterisation and time-lapse analysis of the Widuri field, 65th Mtg., Eur., Assn. Expl. Geophys., Extended Abstracts). Such methods apply separate inversions of the data with some constraint between the results, e.g. a common background model. The time-lapse change is then calculated from the change in
10   inverted parameters. Sarkar et al, 2003, On the Inversion of time-lapse seismic data, 73rd Ann. Internat. Mtg.: Soc. Of Expl. Geophys., 1489-1492, mentions inversion of seismic differences, but provides no detail of the implementation. None of these inversion techniques provide uncertainty bounds on the results.

15   According to a first aspect of the invention, there is provided a method as defined in the appended claim 1.

Further aspects and embodiments of the invention are defined in the other appended claims.
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It is thus possible to provide a technique which permits improved inversion of seismic data representing a physical system. Such a technique may be used to handle errors intrinsic to such data and can provide, for example, probability distributions or uncertainty bounds on the results of inversion.
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For a better understanding of the present invention and in order to show how the same may be carried into effect, preferred embodiments of the invention will now be described, by way of example, with reference to the accompanying drawings in which:

30   Figure 1 is a flow diagram illustrating a method constituting an embodiment of the present invention;
Figures 2a to 2c illustrate a reservoir model;
Figures 3a to 3d illustrate modelled seismic data for first and second surveys and their associated noise characteristics;

Figures 4a to 4c and 5a to 5c illustrate the results of an example of the method of Figure 1;

Figure 6 illustrates the uncertainties in the results illustrated in Figures 4a to 4c and 5a to 5c;

5   Figure 7 is a probability map derived from the results of Figures 5a to 5c; and

Figure 8 is a block schematic diagram of an apparatus for performing the method of Figure 1.

The embodiments of the present invention relate to a method of data inversion that
10   operates directly on seismic difference data, and in particular to the difference between two sets of measured data representing a system in first and second states.   In the embodiments described herein, the inversion method estimates the changes of the elastic material properties of a region of the Earth containing a hydrocarbon reservoir due to production or removal of hydrocarbons.   The techniques can be based on both
15   isotropic and anisotropic models, provided that the reflections may be expressed linearly.   The inversion method gives estimates of the changes in the parameters of the model, in addition to the corresponding uncertainty.   The solution is obtained by combining the information provided by difference data with knowledge obtained prior to inversion.   The solution is therefore more robust and less vulnerable to instabilities.
20   Working on difference data rather than inverting the measured data prior to taking the difference is advantageous with respect to uncertainty estimation in that it allows a correct quantitative statistical treatment of the uncertainties, eliminating the need for qualitative interpretation.

25   To assess the uncertainty of inversion results, the inversion process is cast in a statistical setting.   The solution of an inverse problem is not limited to a single best-fitting set of model parameters, but also characterises the uncertainty of the inversion results.   A Bayesian setting is chosen for the inversion, although other methods, e.g. least squares, can also be used to solve the inversion problem.   In a Bayesian setting it is possible to
30   combine available prior knowledge with the information contained in the measured data.   The solution of a Bayesian inverse problem is represented by the posterior distribution, which addresses all questions of nonuniqueness and uncertainty.   In particular, a Gaussian posterior distribution is completely characterised by a posterior expectation and a posterior covariance.

A model of the earth for the region under consideration is defined, characterised by parameters describing the elastic material properties. The embodiments of the present invention implement a forward modelling operator, the details of which shall now be described.

As described above, an isotropic elastic medium may be fully described using a set of three parameters. The embodiments described below adopt the P-wave velocity $\alpha(\mathbf{x}, t)$, the S-wave velocity $\beta(\mathbf{x}, t)$ and the density $\rho(\mathbf{x}, t)$ as the parameterisation variables, where $\mathbf{x}$ is the lateral location and t is the two-way vertical seismic wave travel time. An alternative parameterisation which may be used takes the acoustic impedance $Z_P = \alpha\rho$, the shear impedance $Z_S = \beta\rho$, and the density ρ as the parameterisation variables instead.

The weak contrast reflectivity function r for PP reflections can be written as

$$r(\mathbf{x},t,\theta) = a_\alpha(\mathbf{x},t,\theta)\frac{\partial}{\partial t}\ln\alpha(\mathbf{x},t) + a_\beta(\mathbf{x},t,\theta)\frac{\partial}{\partial t}\ln\beta(\mathbf{x},t) + a_\rho(\mathbf{x},t,\theta)\frac{\partial}{\partial t}\ln\rho(\mathbf{x},t),$$

where θ is the reflection angle, $a_\alpha = (1 + \tan^2\theta)/2$, $a_\beta = -4(\beta/\alpha)^2\sin^2\theta$, and $a_\rho = (1 - 4(\beta/\alpha)^2\sin^2\theta)/2$. The values of a_α, a_β and a_ρ are defined in accordance with a known prior background model of the region under consideration. For zero-incidence reflections, this reduces to

$$r(\mathbf{x},t,0) = \frac{1}{2}\frac{\partial}{\partial t}\ln Z_P(\mathbf{x},t).$$

The techniques described herein are with reference to PP reflections, but the inversion is equally applicable to other types of reflection, including PS and SS reflections, and in general to all linear expressions of reflectivity.

A model parameter vector $\mathbf{m}$ can be defined as

$$\mathbf{m}(\mathbf{x},t) = [\ln\alpha(\mathbf{x},t), \ln\beta(\mathbf{x},t), \ln\rho(\mathbf{x},t)]^T,$$

where T denotes transpose such that $\mathbf{m}(\mathbf{x},t)$ is a column vector. A further row vector $\mathbf{a}$ is defined as

$$\mathbf{a}(\mathbf{x},t,\theta) = [a_\alpha(\mathbf{x},t,\theta), a_\beta(\mathbf{x},t,\theta), a_\rho(\mathbf{x},t,\theta)]$$

such that the reflectivity function given above may be defined by the dot product

$$r(\mathbf{x},t,\theta) = \mathbf{a}(\mathbf{x},t,\theta) \cdot \frac{\partial}{\partial t}\mathbf{m}(\mathbf{x},t).$$

A discrete representation of the reflectivity r in a given time window for a given set of reflection angles can then be written as a vector

$$\mathbf{r}(\mathbf{x}) = \mathbf{A}(\mathbf{x}) \cdot \frac{\partial}{\partial t}\mathbf{m}(\mathbf{x}),$$

where $\mathbf{A}(\mathbf{x})$ is a sparse matrix defined by $\mathbf{a}(\mathbf{x},t,\theta)$ and $\mathbf{m}(\mathbf{x})$ is a discrete representation of the model parameters in location $\mathbf{x}$.

The seismic data $\mathbf{s}$ can be represented by the convolutional model

$$s(\mathbf{x},t,\theta) = \int w(\tau,\theta)r(\mathbf{x},t-\tau,\theta)d\tau + e(\mathbf{x},t,\theta),$$

where w is the seismic wavelet and e is an error term. The wavelet may be angle-dependent, but independent of the lateral position $\mathbf{x}$. The wavelet is assumed to be stationary within a limited target window. A seismic angle gather at location $\mathbf{x}$ can therefore be written, using all of the above, as

$$\mathbf{s}(\mathbf{x}) = \mathbf{WA}(\mathbf{x})\mathbf{Dm}(\mathbf{x}) + \mathbf{e}(\mathbf{x}),$$

where $\mathbf{W}$ is a matrix representation of the angle-dependent wavelets, $\mathbf{A}(\mathbf{x})$ is the sparse matrix defined by $\mathbf{a}$ above, and $\mathbf{D}$ is merely a differential operator representing partial differentiation (with respect to time) of the material parameters. This may be written in more compact notation by defining the forward modelling operator $\mathbf{G}(\mathbf{x}) = \mathbf{WA}(\mathbf{x})\mathbf{D}$ to give a description of a seismic angle gather at location $\mathbf{x}$ of

$$\mathbf{s} = \mathbf{Gm} + \mathbf{e}.$$

Seismic data is obtained from the region under consideration on two separate occasions, shown at step 1 in the flow diagram of Figure 1. Difference data is formed by subtracting a first (baseline) seismic dataset s_1 from a subsequent (repeat) seismic dataset s_2, shown at step 2 of Figure 1. The method described herein may be applied to both the difference of pre-stack seismic data, and the difference of fully stacked or partially stacked data. The seismic difference data is discrete and is represented on a seismic grid. A seismic difference vector $\mathbf{d}$ is defined as a collection of seismic difference data from a set of grid locations, and is represented as $\mathbf{d} = s_2 - s_1$.

The spatially distributed model parameters are represented on a grid covering the region under consideration, using a model parameter vector **m** containing a collection of the model parameters from a set of grid locations, such as those defined above in terms of

5   the P and S-wave velocities and density. The model grid should cover the region of seismic data collection (the seismic grid), but the two do not need to entirely coincide. In most cases, however, coinciding grids would be a preferable choice. The embodiments described herein refer to coinciding grids. The step of defining a parameterised model is shown at step 3 of Figure 1.

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A model parameter change vector is defined as the difference between the model parameter vector $\mathbf{m}_2$ corresponding to the repeat dataset and the model parameter vector $\mathbf{m}_1$ corresponding to the first dataset, that is $\boldsymbol{\delta} = \mathbf{m}_2 - \mathbf{m}_1$, corresponding to step 4 of Figure 1.

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The model parameter change vector $\boldsymbol{\delta}$ may be defined (for constant **x**), using the parameterisation above, as

$$\delta(\mathbf{x},t) = \left[\ln \frac{\alpha_2(\mathbf{x},t)}{\alpha_1(\mathbf{x},t)}, \ln \frac{\beta_2(\mathbf{x},t)}{\beta_1(\mathbf{x},t)}, \ln \frac{\rho_2(\mathbf{x},t)}{\rho_1(\mathbf{x},t)} \right]^T,$$

where the indexes 1 and 2 refer to the first and second seismic data gathers respectively.

20   For zero-incidence reflections, this reduces to

$$\delta(\mathbf{x},t) = \ln \frac{Z_{P,2}(\mathbf{x},t)}{Z_{P,1}(\mathbf{x},t)},$$

where $Z_{P,i} = \alpha_i \rho_i$ is the acoustic impedance at time i.

A linear relationship between the model parameter change vector and the seismic data

25   difference vector can be written in matrix-vector notation as

$$\mathbf{d} = \mathbf{G}\boldsymbol{\delta} + \mathbf{e},$$

where **G** is the forward modelling operator **G(x)** defined above, and **e** is the error term being related directly to the difference data **d**. The formulation of this inversion problem represents a new approach to the inversion of time-lapse seismic data.

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The error term e is predominantly a consequence of seismic noise which may be characterized with an error expectation vector and an error covariance matrix Σ_e. The error expectation vector is here set to be a zero vector. The error covariance matrix Σ_e specifies the variance for each of the elements in the error term vector e, and the

5   correlations between the different elements therein. The error covariance matrix can be estimated from the seismic difference data in regions not influenced by the production under consideration, which is not possible unless data are differenced. The error expectation and error covariance are determined prior to the inversion.

10   The knowledge and uncertainty about the model parameter change vector δ prior to the inversion are characterized via a model parameter change expectation vector μ_δ and a model parameter change covariance matrix Σ_δ. The model parameter change expectation vector and the model parameter change covariance matrix are specified prior to inversion, and can be determined by analysis of the effects of fluid substitution

15   and pressure changes due to production. In regions not affected by production, the model parameter change expectation is zero (no change is expected). The model change parameter covariance matrix specifies the variance for each of the elements $\ln(\alpha_2/\alpha_1)$, $\ln(\beta_2/\beta_1)$, $\ln(\rho_2/\rho_1)$ in the model parameter change vector, and the correlations therebetween.

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Explicit analytical expressions are calculated for a solution in which the prior model is combined with the information provided by the seismic difference data by forming a posterior distribution, shown at step 5 of Figure 1. The solution is represented via an updated model parameter change expectation vector, step 6 of Figure 1, and an updated

25   model parameter change covariance matrix, step 7 of Figure 1. The updated model parameter change expectation vector provides an estimator of the change in material properties, and thus allows quantitative inferences on the nature of the change to be made. The uncertainty is represented via this updated covariance matrix. The analytical explicit expressions for the solution provide a computationally fast inversion method

30   which allows for the assessment of the likelihood of changes in the physical system, shown at step 8 of Figure 1.

The statistical properties of the model parameters are assumed to be Gaussian, although any other statistical distribution, e.g. the Cauchy distribution, may be used. If each single parameter is assumed to be Gaussian defined by an expectation value and a variance, then the model parameter change vector δ is Gaussian, defined by the model parameter change expectation vector and the model parameter change covariance matrix:

$$\delta \sim N_{n_m}(\mu_\delta, \Sigma_\delta),$$

where n_m is the dimension of the model parameter change vector, and μ_δ and Σ_δ are as defined above.

The statistical properties of the noise are also assumed to be Gaussian. If each single seismic noise sample is assumed to be Gaussian defined by an expectation value and a variance, then a vector of noise samples is Gaussian defined by an error expectation vector and an error covariance matrix as described above:

$$d|\delta \sim N_{n_d}(G\delta, \Sigma_e),$$

where n_d is the dimension of the difference data vector d, and G, δ and Σ_e are as described above.

These two distribution models therefore imply that the distribution for the seismic difference data is also Gaussian:

$$d \sim N_{n_d}(\mu_d, \Sigma_d),$$

with a difference expectation vector $\mu_d = G\mu_\delta$ and difference covariance matrix $\Sigma_d = G\Sigma_\delta G^T + \Sigma_e$.

Suppressing the location parameter x, the joint distribution for the model parameter change vector δ and the seismic difference data vector d is then:

$$\begin{bmatrix} \delta \\ d \end{bmatrix} \sim N_{n_m+n_d}\left(\begin{bmatrix} \mu_\delta \\ \mu_d \end{bmatrix}, \begin{bmatrix} \Sigma_\delta & \Sigma_\delta G^T \\ G\Sigma_\delta & \Sigma_d \end{bmatrix} \right).$$

Under the described Gaussian assumptions for the prior model and the seismic noise, the solution is represented via the multi-Gaussian distribution:

$$\delta|d \sim N_{n_m}(\mu_{\delta|d}, \Sigma_{\delta|d}),$$

defined by the new combined expectation and covariance. In this case, the expectation vector represents an optimal solution for the 4D inversion problem. In Bayesian terminology, this is the posterior distribution with a posterior expectation of

$$\mu_{\delta|d} = \mu_\delta + \Sigma_\delta G^T \Sigma_d^{-1} (d - \mu_d)$$

5   and a posterior covariance of

$$\Sigma_{\delta|d} = \Sigma_\delta - \Sigma_\delta G^T \Sigma_d^{-1} G \Sigma_\delta .$$

The posterior expectation is an optimal estimator for δ calculated from the difference data d, such that

10                              $$\hat{\delta} = \mu_{\delta|d},$$

while the uncertainty is represented by the posterior covariance matrix.

The posterior estimator $\hat{\delta}$ is an estimator for $\ln(\alpha_2/\alpha_1)$, $\ln(\beta_2/\beta_1)$ and $\ln(\rho_2/\rho_1)$. In the poststack case, given zero incidence reflections, $\hat{\delta}$ is an estimator for $\ln(Z_{P,2}/Z_{P,1})$.

15   Therefore $\hat{\delta}$ gives a quantitative estimate of the change in material properties, and the corresponding posterior covariance matrix gives its uncertainty. These two quantities may then be used to infer changes in production in the reservoir, and other related reservoir properties.

20   The method of the above embodiment will now be illustrated by way of example.

A reservoir model is defined by acoustic impedance models for the baseline and repeat seismic surveys, illustrated in Figures 2a and 2b. The reservoir is originally saturated by oil, but during production regions of the reservoir are water-flushed. The drainage is

25   modelled by an increase of the acoustic impedance. The percentage change in acoustic impedance is illustrated in Figure 2c, where the brighter-coloured region indicates the region where the physical properties have changed.

For each of the baseline and repeat models, a seismic forward modelling is performed

30   by convolution. The wavelet is a Ricker wavelet with a 25Hz centre frequency and normalised amplitude. The modelled results s_1 and s_2 are shown in Figures 3a and 3b respectively. A combination of coloured noise e_1 and white noise e_2 of the form

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$\mathbf{e} = \mathbf{s}_1 * \mathbf{e}_1 + \mathbf{e}_2$ was added to the difference data $\mathbf{d}$. The coloured noise represents coherent noise, for example a non-repeatability of the source energy between the baseline and repeat surveys. The white noise is Gaussian with a variance of σ_2^2. The seismic difference was simulated with signal-to-noise ratios of S/N=10^5 and 2, shown in Figures 3c and 3d respectively.

To test the inversion method, a stationary prior distribution is assumed, i.e.

$$\mu_\delta(\mathbf{x}, t) = E\left\{ \ln\left[\frac{Z_{P,2}(\mathbf{x}, t)}{Z_{P,1}(\mathbf{x}, t)} \right] \right\} = 0 ,$$

which corresponds to a constant acoustic impedance between surveys. A stationary covariance function with $\sigma_\delta^2 = 0.039$ is also assumed, corresponding to a 0.95 prior model interval of ±8% acoustic impedance change, which widely includes the modelled increase of about 2-4%. The noise covariance and the wavelet used in the inversion are consistent with the model.

The result of inversion of the difference data with a S/N ratio of 10^5 is shown in Figures 4a to 4c, where Figure 4a is the true model, Figure 4b is a waveform representation of the seismic difference data, and Figure 4c shows the posterior mean solution. With this low noise level the acoustic impedance change is retrieved almost exactly, with low uncertainty.

The result of inversion of the difference data with a S/N ratio of 2 is shown in Figures 5a to 5c, where Figure 5a is the true model, Figure 5b is a waveform representation of the seismic difference data, and Figure 5c shows the posterior mean solution. The solution is smoother and has higher uncertainty that that of Figure 4c.

The uncertainties of the inversion results at CDP 115 are shown in Figure 6a for the low noise case (S/N=10^5), and in Figure 6b for the high noise case (S/N=2). The confidence region in Figure 6b is much wider than in Figure 6a, as would be expected.

Based on the uncertainty bounds of the inversion results, it is possible to provide quantitative probabilities of different reservoir states, e.g. drained, undrained, amount of vertical change of a hydrocarbon contact. The probabilities can be used in risk value

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estimation and commercial decisions. In this example, Figure 7 illustrates a probability density map of drained regions (with S/N=2), where darker colouring indicates a higher probability of having been drained. This corresponds very well to the modelled case of Figure 2c.

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The data inversion methods described above may be embodied in a program for controlling a computer to perform the inversion. The program may be stored on a storage medium, for example hard or floppy discs, CD or DVD-recordable media or flash memory storage products. The program may also be transmitted across a 10 computer network, for example the Internet or a group of computers connected together in a LAN.

The schematic diagram of Figure 8 illustrates a central processing unit (CPU) 13 connected to a read-only memory (ROM) 10 and a random access memory (RAM) 12. 15 The CPU is provided with measured data 14 and model parameters 16 via an input/output mechanism 15. The CPU then performs the inversion on the provided data in accordance with the instructions provided by the program storage (11) (which may be a part of the ROM 10) and provides the output, i.e. the updated model parameters and uncertainties 17, via the input/output mechanism 15. The program itself, or any of the 20 inputs and/or outputs to the system may be provided or transmitted to/from a communications network 18, which may be, for example, the Internet.